N°. VI.

A Letter from Mr. ANDREW ELLICOTT, *to* ROBERT PATTERSON; *in Two Parts.*

Part first contains a number of Astronomical Observations.

Part second contains the Theory and Method of calculating the Aberration of the Stars, the Nutation of the Earth's Axis, and the Semiannual Equation.

PART FIRST.

Philadelphia, April 2d, 1795.

DEAR SIR,

Read April 3, 1795.

I HEREWITH present you with a considerable number of Astronomical Observations, which you will observe were generally made on some very important occasions. —The following immersions, and emersions, of the satellites of Jupiter, were observed at Wilmington on the Delaware, by Messrs. Rittenhouse, Page, Andrews, and Lukens; and at the western observatory by Messrs. Ewing, Madison, Hutchins, and myself, for the purpose of determining the western extension of the state of Pennsylvania.

Immersions observed at the Western observatory in 1784.

Day of the month.	Satellite.	Mean Time.	Observers.	Telescopes.*
July 17.	1	12h 13′ 48″	Ewing,	A
		12 13 20	Madison,	B
		12 13 25	Hutchins,	C
		12 13 25	Ellicott.	D

* A a 4 feet acromatic, B 2½ feet reflector, C a 2 feet reflector, and D a 3 feet acromatic.

Immersions observed at Wilmington in 1784.

Day of the Month.	Satellite.	Mean Time.	Observers.	Telescopes.*
July 1.	1	14h 17′ 33″	Page,	E
		14 17 46	Rittenhouse,	G
		14 17 48	Lukens.	F
July 3.	2	13 18 58	Page,	E
		13 19 12	Rittenhouse,	G
		13 19 02	Lukens.	F
July 8.	1	16 11 10	Page,	E
		16 11 27	Rittenhouse.	G

* G a 4 feet reflector, E a 2 feet reflector, F 3½ feet acromatic, and H a 2 feet reflector.

Aug. 3.

Immersions observed at the Western Observatory.

Day of the Month.	Satellite.	Mean Time.	Observers.	Telescopes.
Aug. 3.	3	8h 54′ 56″	Ewing,	A
		8 55 16	Madison,	B
		8 55 23	Hutchins,	C
		8 55 6	Ellicott.	D
Aug. 9.	1	12 24 20	Ewing,	A
		12 24 25	Madison,	B
		12 24 15	Hutchins,	C
		12 24 31	Ellicott,	D
Aug. 10.	3	12 56 24	Ewing,	A
		12 56 29	Madison,	B
		12 56 24	Hutchins,	C
		12 56 8	Ellicott.	D
Aug. 16.	1	14 18 40	Ewing,	A
		14 18 13	Hutchins,	C
		14 19 1	Ellicott.	D
Aug. 19.	4	12 31 2	Ewing,	A
		12 30 57	Madison,	B
		12 31 15	Ellicott.	D

Immersions observed at Wilmington.

Day of the Month.	Satellite.	Mean Time.	Observers.	Telescopes.
Aug. 3.	3	9h 15′ 17″	Andrews,	H
		9 14 47	Page,	E
		9 15 37	Lukens,	F
		9 15 27	Rittenhouse.	G
Aug. 10.	3	13 16 10	Andrews,	H
		13 16 33	Lukens,	F
		13 16 38	Rittenhouse.	G
Aug. 16.	1	14 38 51	Andrews,	H
		14 38 31	Page,	E
		14 38 37	Lukens,	F
		14 38 39	Rittenhouse.	G
Aug. 19.	4	12 49 36	Andrews,	H
		12 49 46	Page,	E
		12 50 26	Lukens,	F
		12 50 21	Rittenhouse.	G
Aug. 23.	1	16 32 11	Andrews,	H
		16 32 45	Lukens,	F
		16 32 49	Rittenhouse.	G

Emersions observed at the Western Observatory, 1784.

Day of the Month.	Satellite.	Mean Time.	Observers.	Telescopes.
Aug. 27.	1	7h 26′ 0″	Ewing,	A
		7 26 35	Madison,	B
		7 26 15	Ellicott,	D
Aug. 29.	2	12 39 41	Ewing,	A
		12 39 58	Madison,	B
		12 40 21	Hutchins,	C
		12 40 8	Ellicott.	D

Emersions observed at Wilmington, 1784.

Day of the Month.	Satellite.	Mean Time.	Observers.	Telescopes.
Aug. 29.	2	13h 0′ 18″	Andrews,	H
		13 0 10	Page,	F
		12 59 43	Rittenhouse.	G
Sept. 8.	3	8 43 23	Andrews,	H
		8 42 55	Page,	F
		8 42 45	Rittenhouse.	G
Sept. 10.	1	11 36 6	Andrews,	H
		11 36 1	Page,	F
		11 35 48	Rittenhouse.	G

Emersion

Emerſions obſerved at the weſtern Obſervatory.

Day of the Month.	Satellite.	Mean Time.	Obſervers.	Teleſcopes.
Sept. 15.	3	12h 22′ 55″	Ewing,	A
		12 23 2	Madiſon,	B
		12 23 31	Hutchins,	C
		12 22 49	Ellicott.	D
Sept. 19.	1	7 38 56	Ewing,	A
		7 39 9	Madiſon,	B
		7 39 6	Hutchins,	C
		7 39 11	Ellicott.	D

Emerſions obſerved at Wilmington.

Day of the Month.	Satellite.	Mean Time.	Obſervers.	Teleſcopes.
Sept. 15.	3	12h 44′ 15″	Andrews,	H
		12 44 8	Lukens,	F
		12 43 45	Rittenhouſe.	G
Sept. 19.	1	7 59 12	Andrews,	H
		7 58 54	Lukens,	F
		7 59 6	Rittenhouſe.	G

Although the correſponding obſervations only, were admitted in the deciſion, the non-correſponding ones may nevertheleſs be uſeful for fixing the geographical ſituations of other places, where correſponding ones may have been made.

In drawing a concluſion from the foregoing obſervations, it was thought neceſſary to conſider what dependence ought to be placed in each ſatellite; becauſe their different velocities, will give different degrees of certainty. The firſt ſatellite is ſmall, but the rapidity of its motion is much more than a compenſation for this deficiency: Its luſtre is much ſooner loſt, or acquired, than that of the ſecond: On the ſame account, the ſecond is better than the third, and the third than the fourth.—The ſlow motion of the third and fourth ſatellites, will occaſion great uncertainty, if the atmoſphere ſhould be more hazy at one obſervatory, than the other, at the time of obſervation: this is manifeſt from the correſponding obſervations of Auguſt 19th and September 15th, both of which would have been rejected, had they not counteracted each other. The firſt ſatellite, being ſo much ſuperior, on account of certainty, to either of the others, we thought proper to put as much dependence upon it, as upon the others collectively, and that the mean of thoſe reſults, ſhould be deemed the aſtronomical diſtance between the eaſtern and weſtern obſervatories.

The correſponding obſervations on the firſt ſatellite, are thoſe of Auguſt 16th and September 19th.

Diff. of longitude by the correſponding obſervations of Auguſt 16th	20′ 1″ 10‴	Immerſion 1ſt Satellite.
Ditto by do. Sept. 15th.	19 58 30	Emerſion 1ſt Satellite.
Diff. of longitude by 1ſt Satellite.	19 59 50 =	the Mean Longitude.

Diff.

Diff. of longitude by the correſponding obſervations of Auguſt 3d.	20	6″	45‴	Immerſion 3d Satellite.
Do. by Do. of Auguſt 10th	20	7	45	Do. Do.
Do. by Do. of Auguſt 19th	18	57	45	Do. 4th Satellite.
Do. by Do. of Auguſt 29th	20	1	40	Emerſion 2d Satellite.
Do. by Do. of September 15th	20	58	10	Do. 3d Satellite.
Longitude by the 2d 3d and 4th Satellites collectively.	20	2	25	= Mean Longitude.
Do. by the 1ſt Satellite	19	59	50	
	20	1	$7\frac{1}{2}$	Mean.

Hence the diſtance between the obſervatories exceeded 5 degrees of longitude, (being the extent of Pennſylvania weſt from a point on the Delaware,) by 1″ 7‴.5.

After the determination, we completed the ſouthern boundary of Pennſylvania; it being likewiſe the north boundary of Maryland, and a part of Virginia, and which had been carried on ſome years before by Meſſrs. Maſon, and Dixon, the diſtance of 242 miles.*

On the 9th day of June 1785, the following obſervations were made at the weſt end of the above line to trace a meridian north, for the weſtern boundary of Pennſylvania, and the eaſtern boundary of a part of Virginia.

Diff. in time between the paſſage of α Libræ, and β Urſæ Min. over our line.			$0^h\ 12'\ 40''.5$
Right Aſcenſion of β Urſæ Min.	$7^s\ 12^\circ\ 54'\ 6''$		
Do. α Libræ.	7 9 46 0		
Diff.	3 8 6	in time=	0 12 29 .2
Error of the line in time			0 0 11 .3

By the above error of 11″.3 in time, it appears that our line was inclined to the weſt 57″ in ſpace, which was corrected on a baſe of 300 perches.

June 29th about 17 miles north from our firſt ſtation, we corrected our line by the following obſervations.

Diff. in time between the paſſage of α Libræ, and β Urſæ Minoris over our line.			$0^h\ 12'\ 17''$
Right Aſcenſion of β Urſæ Min.	$7^s\ 12^\circ\ 53'\ 50''$		
Do. of α Libræ.	7 9 46 11		
Diff.	0 3 7 39	= in time to	0 12 30
Error of the line in time.			0 0 13 diff.

The

* This line is in the parallel of 39° 43′ 18″ north latitude. My aſſociates in this part of the buſineſs were, Dr. Rittenhouſe, Dr. Ewing, Mr. Andrews, and Mr. Hutchins.

The above difference of 13″ in time, is equal to an angle of 1′ 5″ in ſpace, which in this caſe is the error of our line towards the eaſt, and was corrected on a baſe of 110 perches.

On the 16th of July, diſtant from our firſt poſition 29 miles, we examined the direction of our line by the following obſervations.

Diff. in time between the paſſage of μ Sagit. and γ Draco. over our line. - - - -			$0^h\ 9'\ 20.''5$
Right Aſcenſion of μ Sagit.	$9^s\ 0^\circ\ 14'\ 47''$		
Do. γ Draco -	8 27 54 59		
Diff. - -	2 19 48	= in time to	0 9 19
Error in time - - - - -			0 0 1.5 diff.

From the above obſervation it appears that our direction is ſufficiently accurate, and the ſmall error if it can be called one, is to the eaſt.

September 3d we made the following obſervations to rectify the direction of our line.

Diff. in time between the paſſage of *a* Urſæ Majoris, and γ Cephi. over our line. - - -			$0^h\ 40'\ 26''$
Right Aſcenſion of γ Cephi.	$11^s\ 22^\circ\ 40'\ 53''$		
Do. of *a* Urſæ Maj.	5 12 35 18		
Diff. - -	6 10 5 35		
Deduct 6^s. -	6		
-	0 10 5 35	= in time to	0 40 22
Error in time - - - -			0 0 4 diff.

By this obſervation it appears that our line is directed too much towards the eaſt by an angle of 13″.

Diff. in time between the paſſage of *a* Urſæ Minoris, and ε Urſæ Majoris over our line - - -			$0^h\ 5'\ 8''$
Right Aſcenſion of *a* Urſæ Min.	$0^s\ 12^\circ\ 34'\ 13''$		
Do. of *a* Urſæ Maj.	6 11 7 24		
	6 1 26 49		
Deduct - -	6		
Difference -	0 1 26 49	= in time to	0 5 47
Error in time - - - -			0 0 39 diff.

By this obſervation, it appears that our direction is too much eaſt by an angle of 23″.

Error of the line by α Urſæ Majoris, and γ Cephi.	0 13″
Do. by *a* Urſæ Min. and ε Urſæ Majoris.	0 23
	2)0 36
Mean error towards the eaſt	0 18

This correction of 18″ was made on a baſe of 24 perches.

The ſame night, we alſo took the greateſt deviation of the pole ſtar, (*a* Urſæ Min.) and the error diſcovered in the line by that method did not differ more than 1′ from a mean of the other obſervations.—It is alſo worthy of remark, that we had not corrected for ſomewhat more than 54 miles: from which a concluſion may be drawn very favourable to the method uſed in carrying on the line, otherwiſe the error muſt have been more conſiderable in ſuch a diſtance.

On the ſixth day of October, diſtant from our firſt ſtation 90 miles, the direction of our line was examined by the following obſervations.

Diff. in time between the paſſage of α Urſæ Min. and, ε Urſæ Maj. over our line.			0ʰ 4′ 56″
Right Aſcenſion of α Urſæ Min.	0ˢ 12° 35′ 51″		
Do. of ε Urſæ Maj.	6 11 7 39		
	6 1 28 12		
Deduct	6		
Diff.	0 1 28 12	= in time to	0 5 53
Error in time			0 0 57 diff.

The above error in time by *a* Urſæ Min and ε Urſæ Maj. is equal to an angle of 34″, which was to the weſt. This error was corrected on a baſe of 48 perches.

On the 17th of October, diſtant from our firſt poſition about 100 miles, we examined the direction of our line by the following obſervations.

Diff. in time between the paſſage of γ Capricorn, and β Cephi over our line			0ʰ 2′ 16″
Right Aſcenſion of γ Capri.	10ˢ 22° 3′ 9″		
Do. of β Cephi.	10 21 27 17		
Diff.	0 0 35 52	= in time to	0 2 23
Error of the line in time.			0 0 7 diff.

This error in time, (by thoſe ſtars,) is equal to an angle of 46″ which is to the weſt.

Diff. in time between the paſſage of β Urſæ Maj. and Fomalhout over our line - - -						$0^h\ 2'\ 56''$
Right Aſcenſion of β Urſæ Maj.	5^s	12°	$11'$	$4''$		
Do. of Fomalhout	11	11	26	34		
	6	0	44	30		
Deduct -	6					
Diff. - -	0	0	44	30	= in time to	0 2 58
Error of the line in time - - - -						0 0 2 diff.

This error of 2″ in time, is equal to an angle of 10″ the error of the line towards the weſt.

Diff. in time between the paſſage of α Urſæ Min. and ε Urſæ Maj. over our line - -						$0^h\ 6'\ 34''$
Right Aſcenſion of α Urſæ Min.	0^s	12°	$35'$	$50''$		
Do. of ε Urſæ Maj.	6	11	7	41		
	6	1	28	9		
Deduct -	6					
Diff. - -	0	1	28	9	= in time to	0 5 53
Error of the line in time - - -						0 0 41 diff.

By this laſt obſervation, our direction appears to be inclined to the weſt, by an angle of 25″.

Error of the line by γ Capri. and β Cephi.	0′ 46″
Do. by β Urſæ Maj. and Famalhout,	0 10
Do. by α Urſæ Min. and ε Urſæ Maj.	0 25
	3)1 11
Mean error towards the weſt -	0 $23\frac{1}{3}$

This correction of $23''\frac{1}{3}$ was made on a baſe of 40 perches, which cloſed our operations that ſeaſon.*

The year following, (viz. in 1786,) the line was carried on about $55\frac{1}{3}$ miles to Lake Erie by Andrew Porter, and Alexander Maclain: in that diſtance the direction was not corrected by any obſervations, neither could it appear very neceſſary, when we conſider how trifling, and unimportant all the errors were which had been diſcovered the preceding ſeaſon.—The line was run by

* Dr. Rittenhouſe, Joſeph Nevil, Andrew Porter, and myſelf were concerned in this line. Joſeph Nevil left us about the 21ſt of Auguſt, and Dr. Rittenhouſe about the 17th of September.

by a moſt excellent tranſit inſtrument, made by Mr. Bird, and which had been uſed by Meſſrs Maſon and Dixon, ſome years before in this country.

The magnetic variation was taken in many places on this line, and was found at our firſt ſtation at the end of the parallel of latitude to be - - - 1° 5′

latitude to be		1° 5′	East.
5 miles on the line it was		2 3	
11	Do.	2 10	
14	Do.	1 57	
16½	Do.	1 30	
19	Do.	1 25	
20	Do.	1 12.5	
26	Do.	1 17.5	
29	Do.	1 37	
37	Do.	1 7.5	
44	Do.	0 57	
47	Do.	0 40	
51	Do.	0 57.5	
53	Do.	0 50	
57	Do.	1 2.5	
63¾	Do.	0 57.5	
70	Do.	0 51	
75	Do.	0 27.5	
79	Do.	0 17.5	
90	Do.	0 19.5	
100	Do.	0 25	

The ſtate of Pennſylvania is bounded on the north by the 42° of north latitude. This line extends from a point on the Delaware, (which was fixed by Dr. Rittenhouſe and Captain Holland in the year 1774,) and extends weſt to Lake Erie: it was completed in the years 1786, and 1787. In order to carry on the parallel of latitude with as much expedition, and economy as poſſible, we diſpenſed with the method of tracing a line on the arc of a great circle, and correcting into the parallel, as purſued by Meſſrs Maſon and Dixon, in determining the boundary between this ſtate, and the ſtate of Maryland, and which we followed in completing their line in the year 1784. We commenced our operations by running a guide line weſt, with a ſurveying compaſs from the point mentioned on the

Delaware 20¼ miles, and there corrected by the following zenith distances taken at its western termination by a most excellent sector, constructed, and executed, by Dr. Rittenhouse.*

Face of the Sector East, 1786.

July	21st	Observed Z. distance	α Lyræ	3°	23′	46″.5	S
	22	Do.	do.	3	23	46	S
	23	Do.	α Cygni	2	31	52	N
	24	Do.	do.	2	32	1	N
	25	Do.	do.	2	32	1	N
		Do.	Capella	3	46	55	N
August	5	Do.	α Lyræ	3	23	37	S
		Do.	α Cygni	2	32	5	N

Face of the Sector West, 1786.

July	25th	Observed Z. distance	α Lyræ	3°	24′	31″	S
	26	Do.	Capella	3	45	17	N
	29	Do.	do.	3	45	15.5	N
		Do.	α Cygni	2	31	14.5	N
	31	Do.	do.	2	31	16	N
August	1	Do.	Capella	3	45	16	N
		Do.	α Cygni	2	31	18.5	N
	4	Do.	Capella	3	45	17.5	N
		Do.	α Cygni	2	31	19.5	N

Mean latitude deduced from the above observations	41° 59′ 52″.7
By which it appears that we were too far south by	7.3

The correction being made, the guide line was corrected back to the Delaware, and another guide line carried on west 19¼ miles from the corrected point north of our observatory, at the termination of which the following zenith distances were observed.

Face of the Sector East, 1786.

August	17th	Observed Z. distance	α Lyræ	3°	23′	39″.5	S
	18	Do.	do.	3	23	37.5	S
		Do.	α Cygni	2	32	10.5	N
	19	Do.	α Lyræ	3	23	36.5	S
		Do.	α Cygni	2	32	8	N
	20	Do.	α Capella	3	46	1.5	N

*At this station a number of observations were rejected, on account of their disagreement, which we fortunately discovered was owing to the atmosphere being affected by the numerous fires we kept up to keep off the flies, musketoes, and gnats, which are very troublesome in that part of the country.

† *Note* the letters N. S. signify north and south of the Zenith.

Mean

Face of the Sector West, 1786.

August 20th	Obferved Z. diftance	α Lyræ	3°	24′	22″	S
	Do.	α Cygni	2	31	23	N
21	Do.	Capella	3	45	16	N
	Do.	α Lyræ	3	24	23.5	S
	Do.	α Cygni	2	31	24	N

Mean latitude deduced from the foregoing obfervations 41° 59′ 53″
Hence our obfervatory too far fouth by 7

This correction being made, we proceeded as in the firft cafe, and carried on our guide line 21¼ miles, at the termination of which we obferved the following zenith diftances.

Face of the Sector East, 1786.

September 1ft	Obferved Z. diftance	α Cygni	2°	32′	0″	N
2	Do.	do.	2	32	0	N
3	Do.	α Lyræ	3	23	47	S
6	Do.	do.	3	23	44.5	S
	Do.	α Cygni	2	32	1	N
7	Do.	α Lyræ	3	23	45.5	S
	Do.	α Cygni	2	31	59	N

Face of the Sector West, 1786.

September 8th	Obferved Z. diftance	α Lyræ	3°	24′	31″	S
9	Do.	α Cygni	2	31	13	N
	Do.	α Lyræ	3	24	33	S
	Do.	α Cygni	2	31	15	N
10	Do.	do.	2	31	12	N

Mean latitude deduced from the above obfervations 42° 0′ 3.8″
Too far north by 3.8

The above correction of 3″.8 being laid off, we proceeded as formerly, and carried on our guide line 28¼ miles, and obferved the following Z. diftances at its termination.

Face of the Sector East, 1786.

September 22d	Obferved Z. diftance	α Lyræ	3°	23′	36″	S
	Do.	α Cygni	2	32	16	N
23	Do.	α Lyræ	3	23	34.5	S
	Do.	α Cygni	2	32	12	N
24	Do.	α Lyræ	3	23	35	S
	Do.	α Cygni	2	32	16.5	N

Face

Face of the Sector West, 1786.

September 27th	Observed Z. distance	α Lyræ	3°	24′	23″.5	S
	Do.	α Cygni	2	31	23.5	N
28	Do.	do.	2	31	24	N
29	Do.	α Lyræ	3	24	22.5	S
	Do.	α Cygni	2	31	26.5	N
30	Do.	α Lyræ	3	24	24.5	S
	Do.	α Cygni	2	31	26.5	N

Mean latitude by the above observations - - 41° 59′ 55″.2
Too far south by - - 4 .8

The correction being made and our guide line corrected back, we ceased our operations for that season.

In June the year following we carried on our guide line 19¼ miles and at its termination made the following observations.

Face of the Sector West, 1787.

June 19	Observed Z. distance	Capella	3°	45′	2″.5	N
20	Do.	α Lyræ	3	24	54	S
	Do.	γ Androm.	0	42	35	S
	Do.	Capella	3	45	2	N
	Do.	α Cygni	2	30	55.5	N
21	Do.	α Lyræ	3	24	53.5	S
	Do.	δ Cygni	2	36	30.5	N
	Do.	α Cygni	2	30	56	N
22	Do.	γ Androm.	0	42	35.5	S
	Do.	Capella	3	45	1.5	N
23	Do.	α Lyræ	3	24	50.5	S
	Do.	δ Cygni	2	36	31.5	N
	Do.	α Cygni	2	30	57.5	N
	Do.	Capella	3	45	0.5	N
24	Do.	α Lyræ	3	24	53	S
	Do.	δ Cygni	2	36	30	N
25	Do.	Capella	3	44	59.5	N

Face of the Sector East, 1787.

June 26th	Observed Z. distance	α Lyræ	3°	24′	9″.3	S
	Do.	α Cygni	2	31	37.3	N
	Do.	γ Androm.	0	41	52.5	S
	Do.	Capella	3	45	42	N
28	Do.	α Lyræ*	3	24	7	S
	Do.	δ Cygni	2	37	13	N
	Do.	α Cygni	2	31	39	N
	Do.	Capella	3	45	44	N
	Do.	α Lyræ	3	24	6.5	S

* Note the Zenith distances are entered according to the civil account, and therefore α Lyræ by siderial time gaining 3′ 56″ on mean solar time, was twice on the meridian that day.

June

June 29th	Observed Z. distance	δ Cygni	2° 37′ 16″.5 N
	Do.	α Cygni	2 31 44.2 N
	Do.	γ Androm.	0 41 53.2 S
	Do.	Capella	3 45 44 N

Mean latitude by the foregoing observations 42° 0′ 12.4″
Too far north by - - - 12.4

The above correction being made we carried on our guide line 26$\frac{5}{8}$ miles, and at its termination observed the following Zenith distances.

Face of the Sector West, 1787.

July 7th	Observed Z. distance	γ Androm.	0° 42′ 40″.5 S
	Do.	Capella	3 44 54 N
	Do.	α Lyræ	3 24 47 S
8	Do.	Capella	3 44 52 N
9	Do.	α Lyræ	3 24 48 S
10	Do.	α Cygni	2 31 22 N
	Do.	Capella	3 44 54 N
11	Do.	γ Androm.	0 42 41 S
	Do.	Capella	3 44 53.7 N
	Do.	α Lyræ	3 24 47.5 S
12	Do.	δ Cygni	2 36 33 N
	Do.	α Cygni	2 30 58 N
13	Do.	δ Cygni	2 36 32 N
	Do.	α Cygni	2 31 1 N
	Do.	Capella	3 44 56 N

Face of the Sector East, 1787.

July 13th	Observed Z. distance	α Lyræ	3° 24′ 2″ S
14	Do.	γ Androm.	0 41 53 S
	Do.	Capella	3 45 37.9 N
	Do.	α Lyræ	3 24 1 S
15	Do.	α Cygni	2 31 45 N
	Do.	Capella	3 45 40.3 N
16	Do.	α Lyræ	3 24 2 S
	Do.	δ Cygni	2 37 20.5 N
17	Do.	α Cygni	2 31 45.2 N
	Do.	γ Androm.	0 41 54 S
	Do.	Capella	3 45 41 N
	Do.	δ Cygni	3 37 17.4 N
18	Do.	γ Androm.	0 41 54.2 S
	Do.	Capella	3 45 39 N
	Do.	δ Cygni	2 37 20 N
19	Do.	α Cygni	2 31 41.7 N
	Do.	Capella	3 45 40 N

Mean latitude of our Obſervatory -	42° 0′ 15″
Too far north by - - -	15

The above correction being made, we carried on the guide line 30½ miles, and at its termination obſerved the following Zenith diſtances.

Face of the Sector Weſt, 1787.

Auguſt 7th	Obſerved Z. diſtance	α Lyræ	3°	23′	53″.4	S
	Do.	α Cygni	2	31	55.5	N
8	Do.	Capella	3	45	38.7	N
	Do.	δ Cygni	2	37	30.1	N
	Do.	α Cygni	2	31	57	N
9	Do.	γ Androm.	0	41	49.5	S
10	Do.	α Lyræ	3	23	53.2	S
11	Do.	α Lyræ	3	23	53.8	S
	Do.	δ Cygni	2	37	33.6	N
	Do.	α Cygni	2	31	54.6	N
12	Do.	Capella	3	45	38.6	N
	Do.	α Lyræ	3	23	52.4	S
	Do.	α Cygni	2	31	57.2	N
13	Do.	γ Androm.	0	41	47.5	S
	Do.	Capella	3	45	36.5	N
	Do.	α Lyræ	3	23	51.8	S
	Do.	δ Cygni	2	37	31.3	N
	Do.	α Cygni	2	31	58.4	N
14	Do.	Capella	3	45	41.5	N

Face of the Sector Eaſt, 1787.

Auguſt 14th	Obſerved Z. diſtance	α Lyræ	3°	23′	8″.5	S
	Do.	δ Cygni	2	38	20.7	N
	Do.	α Cygni	2	32	45.6	N
15	Do.	γ Androm.	0	41	0.7	S
	Do.	α Lyræ	3	23	11	S
	Do.	δ Cygni	2	38	22.6	N
16	Do.	γ Androm.	0	41	2	S
	Do.	α Lyræ	3	23	10.5	S
	Do.	δ Cygni	2	38	23.7	N
	Do.	α Cygni	2	32	42.5	N

Mean latitude of the obſervatory - -	41° 59′ 27″.5
Too far ſouth by - -	32 .5

Corrected as formerly, and carried on the guide line 28¼ miles, and obſerved the following Zenith diſtances.

Face

Face of the Sector West, 1787.

August 25th	Observed Z. distance	δ Cygni	2°	36′	38″.3	N
26	Do.	β Medusæ	1	53	12.5	S
	Do.	Capella	3	44	47.9	N
27	Do.	α Lyræ	3	24	45.7	S
	Do.	δ Cygni	2	36	39	N
	Do.	α Cygni	2	31	8.4	N
30	Do.	Capella	3	44	49.2	N
	Do.	α Lyræ	3	24	42	S
	Do.	α Cygni	2	31	6.5	N
31	Do.	γ Androm.	0	42	32.9	S
	Do.	β Meadusæ	1	53	15.9	S
	Do.	α Lyræ	3	24	44.9	S
	Do.	δ Cygni	2	36	41.5	N
	Do.	α Cygni	2	31	10.2	N
Sept. 2d	Do.	β Medusæ	1	53	11.3	S

Face of the Sector East, 1787.

September 2d	Observed Z. distance	α Lyræ	3°	23′	58″.2	S
	Do.	δ Cygni	2	37	24.5	N
	Do.	α Cygni	2	31	55.4	N
3	Do.	γ Androm.	0	41	50.6	S
	Do.	α Lyræ	3	23	59.2	S
	Do.	δ Cygni	2	37	27.5	N
	Do.	α Cygni	2	31	56.6	N
4	Do.	γ Androm.	0	41	47.4	S
	Do.	β Medusæ	1	52	26.8	S
	Do.	Capella	3	45	31.5	N
	Do.	α Lyræ	3	23	57.8	S
	Do.	δ Cygni	2	37	28.8	N
6	Do.	α Lyræ	3	23	58.4	S
	Do.	δ Cygni	2	37	29	N
7	Do.	β Medusæ	1	52	28.5	S
	Do.	α Cygni	2	31	56.4	N
8	Do.	β Medusæ	1	52	27.8	S
	Do.	Capella	3	45	31.1	N
9	Do.	do.	3	45	35	N

Mean latitude of the Observatory - - 42° 0′ 21″.3
Too far north by - - - 21.3

The above correction being made, we carried on the guide line 32½ miles, and observed the following Zenith distances.

Face of the Sector West, 1787.

September 21st	Observed Z. distance	α Lyræ	3°	24′	31″.5	S
	Do.	δ Cygni	2	36	54	N
	Do.	α Cygni	2	31	28.3	N

September 22d	Obſerved Z. diſtance	β Meduſæ	1°	52'	57".9	S
	Do.	Capella	3	45	4.3	N
	Do.	α Lyræ	3	24	31.5	S
	Do.	α Cygni	2	31	28.3	N
23	Do.	γ Androm.	0	42	21.7	S
	Do.	α Lyræ	3	24	32.9	S
	Do.	δ Cygni	2	36	58.5	N
	Do.	α Cygni	2	31	25	N
24	Do.	Capella	3	45	1.2	N
	Do.	δ Cygni	2	36	55.5	N
	Do.	α Cygni	2	31	28.6	N
25	Do.	Capella	3	45	2.5	N
October 2	Do.	γ Androm.	0	42	19.5	S
	Do.	β Meduſæ	1	53	0.3	S

Face of the Sector Eaſt, 1787.

September 25th	Obſerved Z. diſtance	α Lyræ	3°	23'	46".4	S
	Do.	δ Cygni	2	37	45.8	N
	Do.	α Cygni	2	32	14.5	N
26	Do.	α Lyræ	3	23	43.5	S
	Do.	δ Cygni	2	37	42.5	N
	Do.	α Cygni	2	32	14.9	N
27	Do.	α Lyræ	3	23	42.9	S
	Do.	δ Cygni	2	37	41.3	N
28	Do.	δ Cygni	2	37	41.3	N
	Do.	α Cygni	2	32	11.2	N
29	Do.	γ Androm.	0	41	39	S
30	Do.	γ Androm.	0	41	38.7	S
	Do.	β Meduſæ	1	52	12	S
	Do.	Capella	3	45	47.2	N
	Do.	α Lyræ	3	23	46.2	S
	Do.	δ Cygni	2	37	42	N
	Do.	α Cygni	2	32	13	N
October 1	Do.	γ Androm	0	41	35	S
	Do.	β Meduſæ	1	52	11.5	S
	Do.	Capella	3	45	45.6	N

Mean latitude of the Obſervatory - -	42° 0' 10".8
Too far north by - - -	10 .8

Corrected as formerly, and carried on our guide line 32¼ miles, to Lake Erie, and obſerved the following Zenith diſtances.

Face of the Sector Weſt, 1787.

October 8th	Obſerved Z. diſtance	δ Cygni	2°	37'	12".9	N
	Do.	α Cygni	2	31	40.5	N

Oct. 9th

October 9th	Obferved Z. diftance	γ Androm.	0°	42′	1″.9	S
	Do.	β Medufæ	1	52	43.8	S
	Do.	Capella	3	45	18.1	N
10	Do.	β Medufæ	1	52	44.5	S
	Do.	Capella	3	45	13.5	N
	Do.	α Lyræ	3	24	17.4	S
	Do.	δ Cygni	2	37	11.4	N
	Do.	α Cygni	2	31	41.3	N
11	Do.	γ Androm.	0	42	2.4	S
	Do.	β Medufæ	1	52	47	S
	Do.	α Lyræ	3	24	15.3	S
13	Do.	α Cygni	2	31	37	N
14	Do.	β Medufæ	1	52	42.8	S
	Do.	Capella	3	45	13.8	N
	Do.	α Lyræ	3	24	21.5	S
	Do.	δ Cygni	2	37	10.2	N
	Do.	α Cygni	2	31	41.5	N
15	Do.	γ Androm.	0	42	1.6	S
	Do.	β Medufæ	1	52	47.1	S
	Do.	Capella	3	45	17.6	N

Face of the Sector East, 1787.

October 15	Obferved Z. diftance	α Lyræ	2°	23′	34″.7	S
	Do.	δ Cygni	2	37	54.5	N
	Do.	α Cygni	2	32	25.4	N
16	Do.	γ Androm.	0	41	14.2	S
	Do.	β Medufæ	1	52	0.4	S
	Do.	Capella	3	45	58.5	N
17	Do.	β Madufæ	1	51	59.9	S
	Do.	α Lyræ	3	23	34.9	S
	Do.	δ Cygni	2	37	57	N
	Do.	α Cygni	2	32	27.6	N
18	Do.	Capella	3	45	58.2	N
	Do.	α Lyræ	3	23	31.2	S
	Do.	δ Cygni	2	37	55.2	N
	Do.	α Cygni	2	32	24.7	N
19	Do.	γ Androm.	0	41	13.2	S
	Do.	β Medufæ	1	51	58.4	S
	Do.	δ Cygni	2	37	51.1	N
	Do.	α Cygni	2	32	25.9	N
20	Do.	γ Androm.	0	41	13.3	S
	Do.	β Medufæ	1	51	57.4	S

Mean latitude of the Obfervatory by the above obfervations 41° 59′ 58″.7
Too far fouth by - - - 1 .3

The above correction being made, completed the Aftronomical boundaries of the State of Pennfylvania.

 My

My aſſociates in tracing the north boundary of Pennſylvania were Dr. Rittenhouſe, James Clinton, and Simeon De Wit, in the year 1786. The firſt of thoſe gentlemen left us in the beginning of September.—The year following my aſſociates were Andrew Porter, Abraham Hardenberg, and William Morris.

I have omitted the calculations, and given only the reſults, for the following reaſons, *firſt* they would have ſwelled this paper to a great length, *ſecondly* no difficulty can ariſe in making them, to any perſon moderately acquainted with practical aſtronomy, except in thoſe ſmall equations depending upon the effects of aberration and nutation, which from the preſent improved ſtate of this ſcience, have become abſolutely neceſſary; and *thirdly* becauſe I intend concluding this paper, with a ſhort eſſay deſigned to render eaſy ſo much of the calculations, as depend upon the effects of aberration and nutation.

The following emerſions of the 1ſt Satellite of Jupiter were abſerved in Baltimore, in the State of Maryland. The teleſcope which I uſed was acromatic, and magnified about 60 times.

January 1788,	2^d	8^h	$6'$	$23''$	hazy atmoſphere	Mean Time.
	9	10	0	14	very clear	
	18	6	23	57	do.	
	25	8	18	54	do.	

Obſervations made at Georgetown, in the diſtrict of Columbia on the annular eclipſe of the Sun in the year 1791.

The beginning of the eclipſe could not be obſerved, the ſun being below the horizon.

April 2d	18^h	$39'$	$1'.25$	annulas completed	Mean Time.
	18	43	15 .25	annulas broken	
	19	55	37 .75	end of the eclipſe	

From an uncommon undulation in the atmoſphere till towards the end of the eclipſe, I cannot pretend to be certain within two or three ſeconds of the completion, and breaking of the annulas; but the end may be relied on as correct. The lat. of Georgetown is about 38° 55′ N.

In the city of Waſhington lat. 38° 52′ 40″ N. I obſerved the following occultation of Aldebaran by the Moon.

Immerſion	January 1793	21^d	7^h	$55'$	$49''.5$	Apparent Time.
Emerſion		21	9	25	21 .5	

A number

A number of the eclipſes of the firſt Satellite of Jupiter, together with a great proportion of my notes relative to the city of Waſhington, were privately taken from my lodgings in Georgetown, otherwiſe they ſhould have appeared in this paper.

As the city of Waſhington from its ſhortly becoming the permanent ſeat of the government of the United States, muſt be an object of importance, I preſume it will not be unacceptable to give ſome account of the method uſed in laying out the ten miles ſquare, and executing the plan of the city.—Preparative to beginning the ten miles ſquare, a meridian was tracea at Jones's Point on the weſt ſide of the Potomak: from this meridian an angle of 45° was laid off north-weſterly, and a ſtraight line continued in that direction ten miles; from the termination of this line making a right angle with it, a ſtraight line was carried north-eaſterly ten miles: from the termination of this ſecond line, a third making a right angle with it was carried ſouth-eaſterly ten miles; and from the beginning on Jones's Point, a fourth was carried ten miles to the termination of the third. Theſe lines were meaſured with a chain which was examined and corrected daily, and plumbed wherever the ground was uneven, and traced with a tranſit and equal altitude inſtrument which I conſtructed and executed in 1789, and uſed in running the weſtern boundary of the State of New York. This inſtrument was ſimilar to that deſcribed by M. Le Monnier in his preface to the French Hiſtoire Celeſte; except in being accommodated to a firm portable triangular frame. The tranſit and equal altitude inſtrument is of all others the moſt perfect, and beſt calculated for running ſtraight lines, and when the different verifications are carefully attended to, may ſafely be conſidered as abſolutely perfect. The lines of the ten miles ſquare were opened forty feet wide, and a mile-ſtone ſet up at the termination of each mile where the ground would admit of it, and marked with the magnetic variation at that particular ſpot.

In order to execute the plan of the city, a meridional line was drawn through the area intended to be occupied by the capitol, and croſſed at right angles by another line paſſing through the ſame area: theſe lines were continued to the extremities of the city, and became the baſis on which the moſt

conſiderable

conſiderable part of the plan was executed.——I firſt endeavoured to lay off the parallels with a chain; but from its great uncertainty, owing to its expanſion and contraction with heat and cold, and the bending and ſtraightening of the links, was under the neceſſity after making many trials of laying it wholly aſide, and in its place made uſe of wooden meaſuring rods, formed like a carpenter's ſquare: theſe rods were truly graduated, and accommodated with plummets and ſliders, by the due management of which, the meaſurements were always horizontal.——After adopting the uſe of the rods I had but one difficulty for ſome time to contend with, which was the tallies being ſometimes returned erroneous for want of the neceſſary care in the meaſurers. The next difficulty was of a much more ſerious nature; it was the points of interſection of ſome of the leading avenues which fixed the poſition of other ſtreets being moved. Upon making this diſcovery I at firſt ſuſpected that it had been done by ſome perſon, or perſons through inadvertence; but from ſubſequent events am inclined to think it was the effect of deſign. I have mentioned this circumſtance to ſhew the neceſſity of a conſtant attention in thoſe intruſted with the execution of ſuch complicated plans to the poſition, and ſituation of all the leading points.

After the principal avenues were fixed, great part of the work could be examined and corrected with mathematical exactneſs, and the ſmalleſt error in any of the meaſurements detected with certainty.

The following are the the inclinations of ſeveral of the leading avenues to the meridian.

Maſſachuſetts avenue eaſt of 1ſt ſtreet weſt, and North Carolina and Georgia avenues, make an angle with the meridian of - - -	62° 29′ 32″
Virginia avenue eaſtward from the place where the Equeſtrian Statue of General Waſhington is to be placed, makes an angle with the meridian of - -	70 18 5
Pennſylvania and Maryland avenues, eaſt of the capitol, make an angle with the meridian of - -	62 27 00
Kentucky and an avenue not yet named, make an angle with the meridian of : - -	33 00 00
Water ſtreet between 7th and 12th ſtreets weſt, makes an angle with the meridian of - - -	44 49 50
New Jerſey and Delaware avenues, make an angle with the meridian of - - - -	15 43 24

Pennſylvania avenue between the capitol and Preſident's Houſe, and Maryland avenue weſt of the capitol make an angle with the meridian of - -	70° 30' 23'

All the lines of the city in which I have been concerned were traced with the ſame inſtrument which I uſed on the lines of the ten miles ſquare, but as the northern part was not finiſhed when I left that place, I cannot pretend to ſay what method has been ſince purſued.

This paper being already carried to a greater length than I at firſt intended, (but upon looking over my notes I find it is yet ſhort of what was originally deſigned for the ſociety,) I muſt therefore in conſequence of numerous avocations, reſerve the remainder for a future communication, and proceed to the ſubjects of aberration and nutation.

N°. VII.

Of the Aberration of the Stars, Nutation of the Earth's Axis, and Semiannual Equation, by ANDREW ELLICOTT.

PART SECOND.

Of the Aberration of the Stars.

Read April 3, 1795.

THE aberration of the ſtars is their ſmall apparent motion occaſioned by the velocity of the Earth in its orbit bearing a ſenſible proportion to the velocity of light. To give an idea of this effect, ſuppoſe an infinite number of particles of matter moving in the direction of A towards B (Fig. 1 Plate I.) at the ſame time ſuppoſe the tube *a* to be moving towards C and preſerving its paralleliſm; then if the velocity of the tube *a* towards C bears no ſenſible proportion to the velocity of the particles moving from A towards B, a particle which enters the centre of the tube at top will fall upon the centre at the bottom. But if the velocity of the tube towards C bears a ſenſible proportion to the velocity of the particles moving from A towards B, then the particles which fall into

into the centre of the tube at top will not fall upon the centre at the bottom, unlefs the tube ſhould be inclined towards the moving particles like the tube *b*, which inclination muſt be more or lefs as the velocity of the tube in croſſing the direction of the particles, is more or lefs ſenſible when applied to their velocity. Now ſuppoſe theſe particles to be rays of light, iſſuing from a ſtar, the line DC a portion of the Earth's orbit, and the tube *a* a teleſcope, then from the theory it is manifeſt, that if the velocity of the Earth in its orbit, bears a ſenſible proportion to the velocity of light, the teleſcope muſt have a direction which will vary from the true place of the ſtar, in order to bring the light through the viſual axis of the inſtrument.

From the ratio of the velocity of the Earth in its orbit, to the velocity of light, a ſtar may poſſibly appear 20 from its true place, which has alſo been confirmed by celeſtial obſervation, and is the full aberration; but this quantity in declination, and right aſcenſion, will only be had in ſtars particularly ſituated, as in the poles of the ecliptic for declination, and in the ſolſtitial colures for right aſcenſion. A ſtar ſituated in either pole of the ecliptic, will apparently deſcribe a circle round its true place, whoſe radius is 20″; and in the ecliptic apparently vibrate backward and forward in its plane, in a ſtraight line whoſe length is 40″. In whatever figure the ecliptic would be projected when viewed from a ſtar, that ſtar will apparently deſcribe a ſimilar one, which muſt be either a ſtraight line a circle, or an ellipſe.—A ſtraight line if the ſtar is in the ecliptic, a circle if in either pole of the ecliptic, and if in either of the intermediate ſpaces an ellipſe, whoſe ſemitranſverſe will be 20″, and ſemi-conjugate the ſine of the ſtar's latitude, making radius, or the ſine of 90° equal to 20.——ſo far for the theory.

It will be adviſible for thoſe not conſtantly in the habit of making the calculations, to begin by projecting the caſe, which may be done as follows. For an example take β Meduſæ, whoſe longitude is $1^s\ 23^\circ\ 13'$, and latitude $22^\circ\ 28'$ north.—From any ſcale of equal parts take 20, and with that extent for a radius deſcribe the circle ABCD, (Fig. 2 Plate I.) through which at right angles to each other, draw the diameters AC, and BD: let BD be the tranſverſe diameter of the ellipſe. Then for the conjugate ſay

As rad. or fine of 90°	. .	Log.	10.00000
Is to 20 the equal parts contained in rad.		Log.	1.30103
So is the fine of the lat. 22° 28′	-	Log.	9.58223
To 7.6 the equal parts cont. in the femi-conjugate		Log.	0.88326

From the fame fcale of equal parts take 7.6, and from the centre of the circle at E, apply this diftance each way on the diameter AC: fuppofe thofe points to be at F, and G, then will FG, be the conjugate diameter of the ellipfe BFDG apparently difcribed by the ftar. The ellipfe muft be divided fimilar to the ecliptic into figns, &c. to fhew the Sun's place. This divifion muft begin from the longitude of the ftar, for which the projection is made, which in the prefent cafe is 1^s 23° 13′ at the point F.—From the point A in the primitive circle lay off 23° 13′, (the excefs of the ftar's longitude above 1^s,) towards B, to the point *z*: then from the point *z*, draw the occult line *z*1 to the periphery of the ellipfe parallel to AC, and the place of the firft fign will be had—next from the point *z*, in the primitive lay off 30° or one fign each way, and from thofe points, as in the firft cafe, draw parallels to AC, meeting the periphery of the ellipfe, and the pofition of 0^s, and 2^s will be had: In this manner the whole periphery of the ellipfe may be graduated into figns, and degrees if the projection fhould be fufficiently large.

The next requifite is to draw the meridian of the ftar through the centre of the projection. In order to do this, the angle made by the interfection of the circle of the ftar's longitude, with the circle of its right afcenfion, muft be determined; which in the prefent cafe is about 18° 11′: this quantity muft be laid off in the primitive from A to M, towards B*: then from M through the centre of the projection draw MEP cutting the ellipfe in the point *u*, and it will be the meridian required.

From a little confideration it will be eafy to conceive that the effect of aberration will always be found three figns behind the Sun's place—hence the aberration anfwering to 2^s of the Sun's place, muft be eftimated at 11^s——and the occult line E 11,

* It may be obferved for a general rule that when the right afcenfion of the ftar is lefs than 3^s and more than 9^s the meridian muft be laid off from A towards B; when more than 3^s and lefs than 9^s from A towards D.

will be the apparent diſtance of the ſtar from its true place. From 11 draw 11 p perpendicular to the meridian of the ſtar, and that diſtance will be the aberration in right aſcenſion, which is always at right angles to the meridian, and the diſtance Ep, on the meridian will be the effect in declination.—The firſt meaſured on the ſcale by which the projection was made, will give 18″.62, and the latter about 7″.12: But the firſt muſt be reduced to the equator, which may be done various ways, but the moſt expeditious is by multiplying it into the natural ſecant of the ſtar's declination, which will give 24″.34, the effect of aberration in right aſcenſion anſwering to 2^s and 8 of the Sun's place; but with contrary ſigns of application*. If the projection ſhould be large, this method will anſwer for common purpoſes, but when great accuracy is required, the quantities muſt be determined by calculation. For this purpoſe, draw the diameter RS, at right angles to the meridian, and cutting the ellipſe in the point m. Then in the right angled ſpherical triangle mEu †, right angled at E, it will be neceſſary to find the arcs uF, Fm, and the angles muE, umE.—The angle muE muſt be firſt obtained by ſolving the right angled ſpherical triangle EFu, right angled at F.—the arc EF being 22 28′ the latitude of the ſtar, and the angle FEu 18° 11′. From theſe data, the angle FuE will be found 73° 21′—the arc Fu 7° 9—the angle FmE 28° 31′, and the arc Fm 49° 21.—To find the aberration in right aſcenſion anſwering to 2^s and 8,——3^s and 9^s,——4^s and 10^s &c. in the projection, add to the log. ſine of the angle EuF = 73° 21′ the log. of 20, and from that ſum deduct 10 for a conſtant log. to the conſtant log. add ſeparately, the log. ſines of the arcs u2, u3, u4, &c. from each of theſe ſums, deduct 10, and the numbers anſwering to the log. remainders, will be the values of 2h, 3i, 10t, &c. Each of thoſe values being multiplied by the natural ſecant of the ſtar's declination, will give the effect in right aſcenſion, as in the following examples.

* The algebraic ſigns of + plus, and — minus.

† An ellipſe may be conſidered a circle in the orthographical projection of the ſphere, the ſemi-conjugate being the co-ſine of the circle's elevation above the primitive.

Angle

Angle E u F 73° 21′ Log. S.	9.98141		
Add 20 Log.	1.30103		
	1.28244	Conſtant Log.	
Conſtant - Log.	1.28244		
Add arc u 2^s = 14° 41′ Log. S.	9.40394		
	0.68638	= 4.86 = 2^s h	for 2^s and 8^s in the projection
Multiplied by nat. Sec^t.	40° 6′ =	× 1.307 = 6″.35	but 5^s and 11^s of the Sun's Longitude
Conſtant - Log.	1.28244		
Add arc u 3^s = 44° 41′ Log. S.	9.84707		
	1.12951	= 13.48 = 3^s i	for 3^s and 9^s in the projection
Multiplied by nat. Sec^t.	40° 6′ =	× 1.307 = 17″.62	but 6^s and 6^s of the Sun's Longitude
Conſtant - Log.	1.28244		
Add arc u 4^s = 74° 41′ Log. S.	9.98429		
	1.26673	= 18.48 = 10^s t	for 4^s and 10^s in the projection
Multiplied by nat. Sec^t.	40° 6′ =	× 1.307 = 24″.15	but 7^s and 1^s of the Sun's Longitude
Conſtant - Log.	1.28244		
Add arc u 5^s = 104° 41′ Log. S.	9.98750		
	1.26994	= 18.62 = 11^s p	for 5^s and 11^s in the projection
Multiplied by nat. Sec^t.	40° 6′ =	× 1.307 = 24″.34	but 8^s and 2^s of the Sun's Longitude
Conſtant - Log.	1.28244		
Add arc u 6^s = 134° 41′ Log. S.	9.85924	= 13.85 = 0^s y	for 6^s and 0^s in the projection
Muitiplied by nat. Sec^t.	40° 6′ =	× 1.307 = 18″.10	but 9^s and 3^s of the Sun's Longitude
Conſtant - Log.	1.28244		
Add arc u 7^s = 164° 41′ Log. S.	9.44862	= 5.38 = 1^s x	for 7^s and 1^s in the projection
Multiplied by nat. Sec^t.	40° 6′ =	× 1.307 = 7″.03	but 10^s and 4^s of the Sun's Longitude.

In this manner the calculations may be expeditiouſly made for any degree of the Sun's place in the ecliptic.

The aberration in right aſcenſion is additive, when a point 3^s behind the Sun's longitude falls on the left ſide of the meridian of the ſtar; the right aſcenſion, or point M, being held from you; but negative when the point falls on the contrary ſide of the meridian.

The foregoing equations when tabled will ſtand as follows:

Sun's

Sun's Long.				Sun's Long.
0^s	—	17″.62	+	6^s
1		24.15		7
2		24.34		8
3		18.10		9
4		7.03		10
5	+	6.35	—	11
6		17.62		0

To obtain the aberration in declination, the angle E*mu* is to be used in the same manner the angle E*um* was in the case of right ascension; and the perpendiculars $3^s n$, $2^s o$, $1^s s$, $0^s v$, and $11^s r$, let fall upon the diameter at right angles to the meridian of the star, will be the equations required.

Angle E*mu* 28° 31′	Log. S.	9.67889		
Add 20 -	Log.	1.30103		
		0.97992	Constant Log.	
Constant -	Log.	0.97992		
Add arc *m* 3^s 11° 49′	Log. S.	9.31129		
		0.29121	= 1″.96 = 3 *n*	for 3^s and 9^s in the projection but 6^s and 0^s of the Sun's Longitude
Constant -	Log.	0.97992		
Add arc *m* 2^s = 41° 49′	Log. S.	9.82396		
		0.80388	= 6″.36 = 2 *o*	for 2^s and 8^s in the projection but 5^s and 11^s of the Sun's Longitude
Constant -	Log.	0.97992		
Add arc *m* 1^s = 71° 49′	Log. S.	9.97775		
		0.95767	= 9″.07 = 1 *s*	for 1^s and 7^s in the projection but 4^s and 10^s of the Sun's Longitude
Constant -	Log.	0.97992		
Add arc *m* 0^s = 101° 49′	Log. S.	9.99070		
		0.97062	= 9″.35 = 0 *r*	for 0^s and 6^s in the projection but 3^s and 9^s of the Sun's Longitude
Constant -	Log.	0.97992		
Add arc *m* 11^s = 131° 49′	Log. S.	9.87232		
		0.85225	= 7″.12 = 11 *r*	for 11^s and 5^s in the projection but 2^s and 8^s of the Sun's Longitude
Constant -	Log.	0.97992		
Add arc *m* 10^s = 161° 49′	Log. S.	9.49424		
		0.47416	= 2″.98 = 10 *r*	for 10^s and 4^s in the projection but 1^s and 7^s of the Sun's Longitude.

The

The aberration in declination is negative, when a point 3^s behind the Sun's longitude, falls on the ſame ſide of a diameter at right angles to the meridian of the ſtar, with the ſtar's right aſcenſion or point M; but the contrary is to be obſerved when the point falls on the oppoſite ſide. The foregoing equations when tabled will ſtand as below.

Sun's Long.	0^s +	1″.96	—	6^s	Sun's Long.
	1 —	2 .98	+	7	
	2	7 .12		8	
	3	9 .35		9	
	4	9 .07		10	
	5	6 .36		11	
	6	1 .96		0	

Of Nutation.

THE nutation or libratory motion of the Earth's axis is occaſioned by the inclination of the Moon's orbit to the ecliptic, and the retrograde revolution of her nodes; which is performed in about 18 years and 7 months. On which account the action of the Moon on the equatorial, or longer diameter of the Earth, is not uniform, and muſt therefore from the principles of gravity produce a motion in the Earth's axis, which will be apparently in the ſtars. For the completion of this diſcovery, we are indebted to the very laborious, and ingenious Dr. Bradley.* This effect of the Moon has been ſettled by a ſeries of accurate obſervations, and therefore not to be conſidered as a ſpeculative argument in favour of the Newtonian Philoſophy; but an abſolute confirmation of it.

It muſt be evident from the theory, that the poles of the equator will complete a retrograde revolution about the mean poles, in the ſame period which completes a revolution of the Moon's nodes: But as the action of the Moon on the equatorial diameter of the Earth, will be ſomewhat varied in different ſituations of her nodes, this revolution of the poles will not be performed in a circle, but a ſmall ellipſe, with the tranſverſe diameter lying in the ſolſtitial colure, and amounting to 19 .1,

* Vide his paper upon this ſubject in Vol. 45, N° 1. of the Tranſactions of the Royal Society.

and

and the conjugate in the equinoctial colure, and has been ſettled at 14 .2.—Let P (fig. 3 Plate I.) repreſent the mean northern axis of the earth.—AB a portion of the ſolſtitial colure, and CD a portion of the equinoctial colure.—From P, each way on AB, lay off 9.55, ſuppoſe to F, and G, then from the ſame point P, lay off each way 7.1, ſuppoſe to E and H, through FHGE deſcribe an ellipſe, and it will repreſent the path deſcribed by the axis of the Earth. When the Moon's aſcending node is in the beginning of ♈, the northern axis of the Earth will be at F, when the ſame node is in the beginning of ♎, the pole will be at H.—conſtantly 3ˢ before the Moon's aſcending node. From theſe elements it is evident, that the obliquity of the ecliptic muſt be ſubject to a periodical change, being greater by 19″.1, when the Moon's aſcending node is in ♈, than when in ♎: and the equinoctial points will alſo be ſubject to an equation, which will be a maximum when the Moon's aſcending node is in the beginning of ♋ and ♑; this equation is common to all the ſtars.

As in the caſe of aberration, it will be proper to make the calculations from an orthographical projection.—From any ſcale of equal parts take 9.55, and with that diſtance for a radius deſcribe a circle, which divide into twelve equal parts for the ſigns in right aſcenſion; which deſignate by numerical letters; (as in Fig. 4, Plate I.) join III, and IX with a diameter, to repreſent a portion of the ſolſtitial colure; and O, and VI, for a portion of the equinoctial colure, from the centre C, towards O, and VI, lay off 7.1, and deſignate thoſe points by o, and 6, then through the points o, III, 6 and IX, deſcribe an ellipſe; which muſt be divided ſimilar to the primitive anſwering to the places of the Moon's aſcending node; and to prevent confuſion in the explanation, it will be convenient to deſignate the ſigns by figures.

To apply β Meduſæ to the projection,* lay off its right aſcenſion 1ˢ 13° 43' from oˢ, in the primitive, according to the order of the ſigns to the point A, then from A, through the centre C, draw the diameter AB for the meridian of the ſtar; which croſs at right angles by the diameter DE. This being done,

* This projection will ſerve for any ſtar; on which account it differs from a projection for aberration.

ſuppoſe

ſuppoſe the place of the Moon's aſcending node to be at 1, and the pole of the Earth being 3^s before the Moon's aſcending node, will be at 4^s in the ellipſe: and the occult line 4 *a*, at right angles to the meridian of the ſtar, will be the nutation in right aſcenſion anſwering to 1^s, and 7^s, of the longitude of the Moon's aſcending node, but with contrary ſigns of application. The diſtance C *a*, or the occult line 4*b*, in the direction of the meridian will be the nutation in declination. The diſtance 4 *a*, meaſured on the ſame ſcale by which the projection is made, will give 8 .44, and the diſtance 4*b* will give 3 .15: But the firſt muſt be reduced to the equator, which is moſt conveniently done by multiplying it by the natural tangent of the ſtar's declination.

When great accuracy is required, recourſe muſt be had to calculation, which may be done in the ſame manner as purſued in aberration. It has already been obſerved that an ellipſe may be conſidered as a circle in the orthographical projection of the ſphere, and therefore the arc C *o*, which is the meaſure of the angle C 3 *o*, will be had by adding 10, to the log. of 7.1, and from that ſum deducting the log. of 9.55 the remainder will be the log. ſine of the arc C *o* which will be about 48° 2′. Then in the right angled ſpherical triangles C *o g*, and C *o e*, right angled at *o*, it is required to find the angles C *g o*, C *e o*, and the axis *o g*, *o e*, the angle *o* C *g* being the right aſcenſion of the ſtar, and the angle *o* C *e* its complement, and therefore both given. The angle C *g o* will be 62° 28′, the arc *o g* = 35° 25, the angle C *e o* = 61° 6′, and the arc *o e* = 37° 52. Theſe being the neceſſary requiſites, the nutation in right aſcenſion will be had as follows

To - 9 55	-	Log.	0.98000	
Add angle C *g o* = 62° 28′	-	Log. S.	9.94780	
			10.92780	
Deduct -	-	-	10	
			0.92780	Conſtant Log.
Conſtant -	-	Log.	0.92780	
Add arc *g* 1 = 5° 25′	-	Log. S.	8.97496	
			9.90276	

As radius cannot be deducted, the number will be a fraction, and the index of the log. being 9, the log. fraction will be - - $\bar{1}.90276 = .80 = 1\,m$ { for 1^s and 7^s in the projection
Multiply by nat. tang^t. of $40° 6'$ = the star's declination = $\times\ .842 = .67$ { but 10^s and 4^s of the Longitude of Moon's node

Constant - -	Log.	0.92780	
4 Add arc $g\,o = 35° 25'$ -	Log. S.	9.76307	
		$0.69087 = 4.91 = 0\,n$	for 0^s and 6^s in the projection
Multiply by - - -		$\times\ .842 = 4''.13$	but 9^s and 3^s of the Long. of Moon's node
Constant - -	Log.	0.92780	
Add arc $g\,11 = 65° 25'$ -	Log. S.	9.95873	
		$0.88653 = 7.70 = 11\,p$	for 11^s and 5^s in the projection
Multiply by - - -		$\times\ .842 = 6''.48$	but 8^s and 2^s of the Long. of Moon's node
Constant - -	Log.	0.92780	
Add arc $g\,10 = 95° 25'$ -	Log. S.	9.99850	
		$0.92630 = 8.44 = 10\,q$	for 10^s and 4^s in the projection
Multiplied by - - -		$\times\ .842 = 7''.11$	but 7^s and 1^s of the Long. of Moon's node
Constant - -	Log.	0.92780	
Add arc $g\,9 = 125° 25'$ -	Log. S.	9.91114	
		$0.83894 = 6.40 = 9\,r$	for 9^s and 3^s in the projection
Multiply by - - -		$\times\ .842 = 5''.81$	but 6^s and 0^s of the Long. of Moon's node
Constant - -	Log.	0.92780	
Add arc $g\,8 = 155° 25'$ -	Log. S.	9.61911	
		$0.54691 = 3.55 = 8\,s$	for 8^s and 2^s in the projection
Multiply by - - -		$\times\ .842 = 2''.96$	but 5^s and 11^s of the Long. of Moon's node

In

In applying the nutation in right afcenfion, obferve this general rule, that when a point 3 before the longitude of the Moon's afcending node, falls on the right fide of the meridian of the ftar, the point A or right afcenfion being held from you, the nutation will be pofitive for ftars having north declination, but negative for fouth:—the contrary is to be obferved when a point 3^s before the Moon's afcending node, falls on the left fide of the meridian. Agreeably to thefe directions, the foregoing equations when tabled will ftand as follows.

Longitude of Moon's Afcending node				Longitude of Moon's Afcending node
0^s	—	$5''.81$	+	6^s
1		7.11		7
2		6.48		8
3		4.13		9
4		0.67		10
5	+	2.96	—	11
6		5.81		0

The next equation is that of the equinoctial points, which is common to all the ftars, and occafioned by the poles of the Earth inclining to, and receding from the celeftial equator.—Suppofe the Moon's afcending node to be at 9^s, then the pole of the Earth will be at o in the ellipfe, and the diftance C *o* will be its inclination towards ♈.—This inclination for any point in the ellipfe will be a perpendicular let fall upon the tranfverfe axis, which will be to the alteration of the equinoctial points, as the tangent of the obliquity of the ecliptic, is to radius;—hence thefe deviations from the tranfverfe axis of the ellipfe being multiplied by the nat. co-tangent of the obliquity of the ecliptic, will give the equations required.

The quantity C *o* = 7.1 $\left\{\right.$ for 0^s and 6^s in the projection but
Mult. by nat. Co-tang^t of $23^\circ\ 28'$ = × 2.3 = $16''.3$ $\left.\right.$ for 3^s and 9^s of the long. of ☽'s node.

For any other points in the ellipfe add the log. of 9.55, to the log. fine of the arc C *o*, and from that fum deduct 10 for a conftant log. to which add the log. fine of any arc from 3, or 9, and from that fum deduct 10, the remainder will be the log. of a perpendicular let fall from the termination of that arc to the tranfverfe axis.

To	9.55	Log.	0.98000	
Add arc $Co = 48^\circ\ 2' =$	-	Log.	9.87130	
			0.85130	Conftant Log.
2 Conftant	-	Log.	0.85130	
Add arc $3^s\ 1^s = 60^\circ$	-	Log. S.	9.93753	
			$0.78883 = 6.15 = 1c$	for $1^s\ 5^s\ 7^s$ and 11^s in the projection but
Mult. by nat. Co-tang^t of $23^\circ\ 28' =$			$\times\ 2.3 = 14''.14$	for $2^s\ 4^s\ 8^s$ and 10^s of the Long. of Moon's node
Conftant	-	Log.	0.85130	
Add arc $3^s\ 2^s = 30^\circ$	-	Log. S.	9.69897	
			$0.55027 = 3.55 = 2f$	for $2^s\ 4^s\ 8^s$ and 10^s in the projection but
Mult. by nat. Co-tang^t of $23^\circ\ 23'' =$			$\times\ 2.3 = 8''.16$	for $1^s\ 5^s\ 7^s$ and 11^s of the Long. of Moon's node

Thefe equations are additive when a point 3^s before the longitude of the Moon's afcending node falls on the fame fide of the tranfverfe axis with 0^s, but the contrary when the point falls on the other fide, and when tabled will ftand as below.

Longitude of the Moon's Afcending node				Longitude of the Moon's Afcending node
0^s	—	$0''.0$	+	6^s
1		8.16		7
2		14.14		8
3		16.33		9
4		14.14		10
5		8.16		11
6	+	0 0	—	0

As

As theſe equations are neceſſary in every caſe relative to the right aſcenſion of the ſtars, (and common to them all,) it will be found very convenient for thoſe concerned in aſtronomical reſearches to make out a table for every degree of the quadrant.

For the nutation in declination, proceed as follows.

To the angle $Ceo = 61° 6'$		Log. S.	9.94224		
Add	9.55	Log.	0.98000		
			0.92224	Conſtant Log.	
Conſtant		Log.	0.92224		
Add arc $e\ 11 =$	7° 52′	Log. S.	9.13630		
			0.05854	$= 1''.14 = 11\ d$	for 11ˢ and 5ˢ in the projection but 8ˢ and 2ˢ of the Long. of Moon's node
Conſtant		Log.	0.92224		
Add arc $eo =$	37° 52′	Log. S.	9.78804		
			0.71028	$= 5''.13 = 0\ v$	for 0ˢ and 6ˢ in the projection but for 9ˢ and 3ˢ of the Long. of Moon's node
Conſtant		Log.	0.92224		
Add arc $e\ 1 =$	67° 52′	Log. S.	9.96676		
			0.88900	$= 7''.75 = 1\ t$	for 1ˢ and 7ˢ in the projection but for 4ˢ and 10ˢ of the Long. of the Moon's node
Conſtant		Log.	0.92224		
Add arc $e\ 2 =$	97° 52′	Log S.	9.99589		
			0.91813	$= 8''.28 = 8\ u$	for 2ˢ and 8ˢ in the projection but for 5ˢ and 11ˢ of the Long. of Moon's node
Conſtant		Log.	0.92224		
Add arc $e\ 3 =$	127° 52′	Log. S.	9.89732		
			0.81956	$= 6''.60 = 9\ x$	for 3ˢ and 9ˢ of the projection but for 6ˢ and 0ˢ of the Long. of Moon's node
Conſtant		Log.	0.92224		
Add arc $e\ 4 =$	157° 52′	Log. S.	9.57607		
			0.49831	$= 3''.15 = 10\ y$	for 4ˢ and 10ˢ of the projection but for 7ˢ and 1ˢ of the Long. of Moon's node.

In applying the equations for nutation in declination obſerve, that when a point 3^s before the longitude of the Moon's aſcending node falls on the ſame ſide of a diameter at right angles to the meridian of the ſtar with its point of right aſcenſion, the nutation will be additive for ſtars having north declination, but negative for thoſe having ſouth declination; the contrary is to be obſerved when a point 3^s before the longitude of the Moon's aſcending node falls on the other ſide of the diameter. The above equations for nutation in declination will be properly expreſſed in the following table.

Longitude of the Moon's Aſcending Node				Longitude of the Moon's Aſcending Node
0^s	+	6″.60	—	6^s
1		3.15		7
2	—	1.14	+	8
3		5.13		9
4		7.75		10
5		8.28		11
6		6.60		0

The foregoing calculations as combined with the projections, may be rendered ſomewhat more ſimple, by numbering the ſigns of the Sun's place in the ellipſe for aberration 3^s ſhort of the true ſigns; and the ſigns for the place of the Moon's aſcending node in the ellipſe for nutation 3^s forward, by which the calculations will coincide with the ſigns for which they were made, and ſo much of the rules for the application of the equations as depend upon a point 3^s behind the place of the Sun for aberration, and 3^s before the place of the Moon's aſcending node for nutation, will become unneceſſary.

There is yet one other equation which, in very nice operations, ſuch as determining the lengths of meridians, &c. may require ſome attention. It is the effect of the inequality of the action of the Sun between the ſolſtices and equinoxes, on the equatorial diameter of the earth, by which the poles are carried annually, twice round the mean poles in a ſmall circle, whoſe diameter is 1″. By which the equinoctial points, the obliquity of the ecliptic, the right aſcenſion, and declination of the ſtars, are affected in a ſmall degree. The maximum of the alteration

teration of the equinoctial points amounts $1''.15$ or the $\frac{1}{13}$ of a second in time. The obliquity of the ecliptic is greater by $1''$, when the Sun is in the equinoxes, than in the folftices. The right afcenfion of the ftars will be infenfibly affected, unlefs the declinations fhould be very great: the declination of 88° 6' will produce but $1''$ in time, and 81° 15' but $\frac{1}{4}$ of a fecond. From the theory the apparent diftance of every ftar from the pole of the equator will be fubject to a variation of $1''$ twice a year, and there being but three months between the greateft inclination, and reclination, it will fenfibly affect the obfervations made with a good 8 feet zenith fector.—For a further explanation, and in aid to the calculations, take from any fcale of equal parts 5, with that diftance for a radius defcribe a circle, which divide into 12 equal parts for figns, (fee Fig. 5. Plate I.) From what has been already obferved it follows that, when the Sun is at o, the pole will be at 3, when the Sun is at 3, the pole will be at 9, and when the Sun is at 6, the pole will be again at o. For an example: Suppofe it fhould be required to find the effect of the femi-annual equation in declination for β Medufæ, anfwering to 3^s of the Sun's place—lay off 1^s 13° 43', the right afcenfion of β Medufæ from o^s, to M; from M, through the centre C, draw the meridian MD; at right angles to which, draw the diameter EF. Then from the theory, whilft the Sun is advancing 3^s, the pole will advance 6^s, and therefore be at 9^s: and the diftance $9\,m$, in the direction of the meridian, will be the quantity required, and when applied to the fcale, will be .34. —this quantity may be readily calculated, being to the fine of arc 9 F = 43° 43' as .5 is to radius, therefore

To .5 - -	Log.	$\bar{1}.69897$
Add 43° 43' -	Log. S.	9.83954
		9.53851

As radius cannot be deducted, the log. muft be expreffed $\bar{1}.53851 = .34 = 9\,m$: in this mannner the calculations may be made for any other points in the circle, and the quantities will be additive to the declination of a northern ftar; when the pole is on the fame fide of a diameter at right angles to the meridian with the point M, of the ftar's right afcenfion; but negative for a fouthern ftar;—the contrary is to be obferved when the pole is on the other fide of the diameter.

The

The following table by attending to the direction will anſwer for all ſtars.

	Argument. From twice the Sun's Longitude take the ſtar's right aſcenſion.					
If the difference be leſs than 6^s add for northern ſtars, but ſubtract for ſouthern.	$0^s\ 0^\circ$	$6^s\ 0^\circ$	$+0''.0$—	$6^s\ 0^\circ$	$0^s\ 0^\circ$	If the difference be more than 6^s ſubtract for northern ſtars, but add for ſouthern.
	15	15	0.13	15	15	
	1 0	5 0	0.25	7 0	11 0	
	15	15	0.35	15	15	
	2 0	4 0	0.43	8 0	10 0	
	15	15	0.48	15	15	
	3 0	3 0	0.50	9 0	9 0	

For an example of the application of the foregoing equations, let it be required to find the right aſcenſion, and declination of β Meduſæ for June 22d 1795; the Sun's longitude being 3^s and the longitude of the Moon's aſcending node 4^s.

Right aſcenſion of β Meduſæ the beginning of 1780.	$1^s\ 13^\circ\ 29'\ 7''.0$
Annual variation for 15 years	+ 14 22.9
Do for June 22d. - -	+ 27.0
Mean right aſcenſion - -	1 13 43 56.9
Aberration - -	— 9.35
Nutation - -	— 0.67
Equation of the equinoctial points	— 14.14
True right aſcenſion -	1 13 43 32.74

Declination of β Meduſæ the beginning of 1780.	$40^\circ\ 5'\ 37''.0$ N
Annual variation for 15 years	+ 3 39.45
Do for June 22d	+ 6.96
Mean declination - -	40 9 23.41
Aberration - -	— 9.35
Nutation - -	— 7.75
Semi-annual equation -	— .34
True declination - -	40 9 5.97

I am, Sir,

Your real Friend,

ANDREW ELLICOTT.

To Robert Patterſon, A. M.

A Letter

Fig. 2. P. 54.

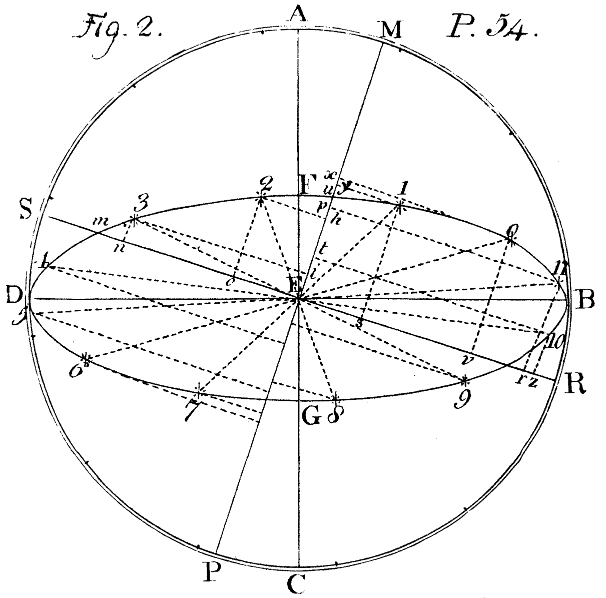

Fig. 1. P. 51.

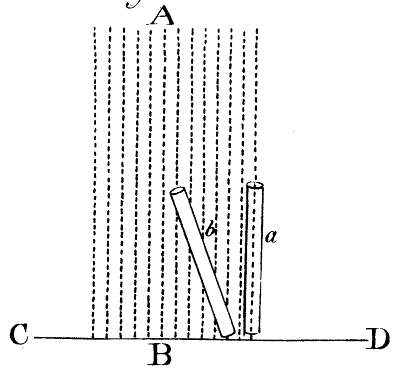

Fig. Θ P. 27.

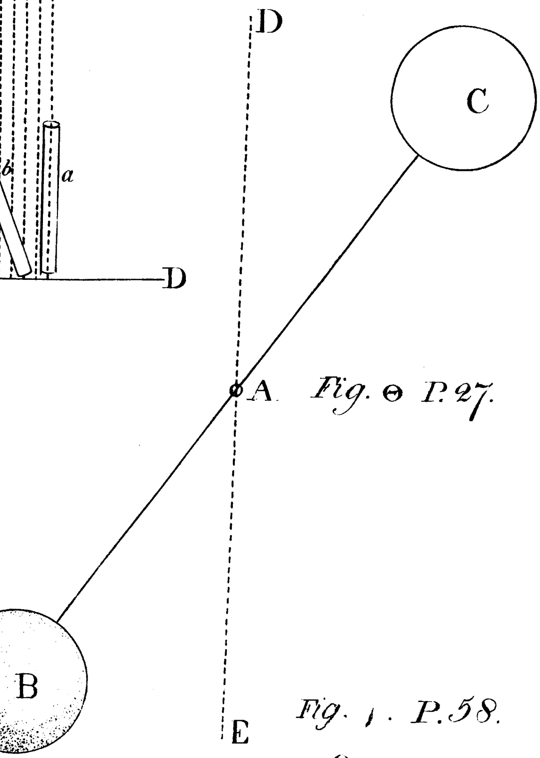

Fig. 3. P. 58.

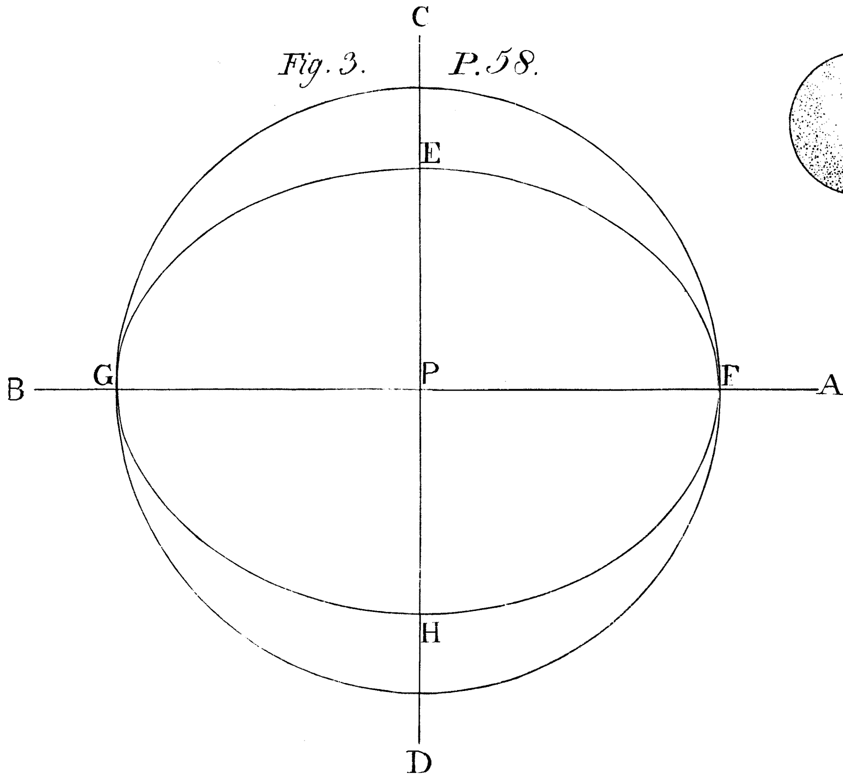

Fig. 4. P. 58.

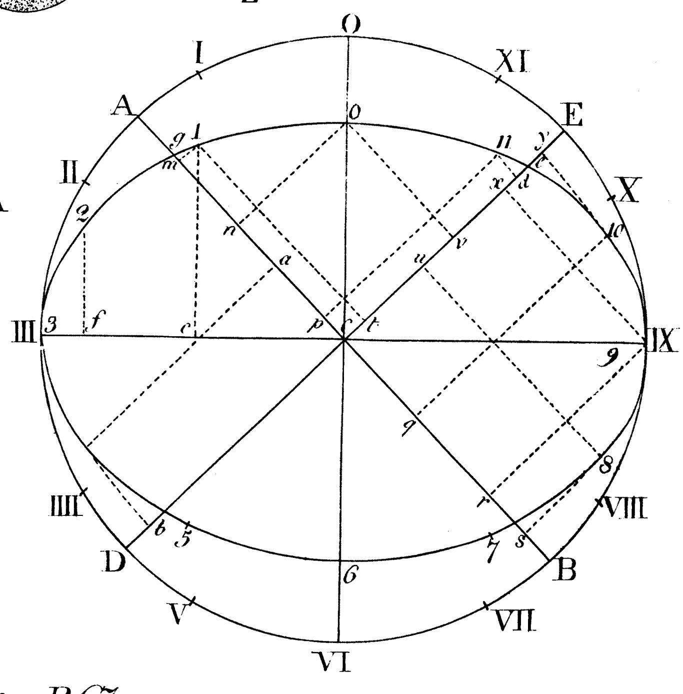

Fig. 5. P. 65.

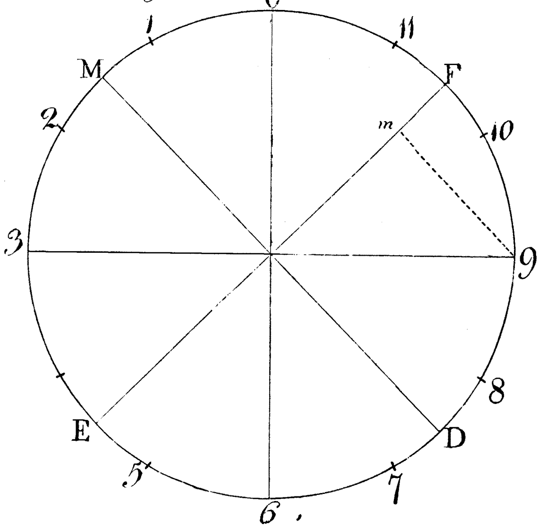

Fig. 9. P. 67.